Atomic Landau-Zener Tunneling and Wannier-Stark Ladders in Optical Potentials

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We calculate the quantum motion of ultracold atoms in an accelerating optical potential, and show how they may be used to observe Landau-Zener tunneling and Wannier-Stark ladders, two fundamental quantum effects in solid state physics. The optical potential is spatially periodic, yielding an energy spectrum of Bloch bands for the atoms. The acceleration provides an inertial force in the moving frame, emulating an electric force on Bloch electrons. [S0031-9007(96)00402-4]

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Quantum transport is a very important subject in solid state physics and electronics applications. Two fundamental quantum effects, Landau-Zener (LZ) tunneling and Wannier-Stark (WS) ladders, occur in a system of electrons moving in a periodic potential and driven by an electric field. Because of complications such as impurities, lattice vibrations, and multiparticle interactions, clean observations of these effects have been difficult. In this Letter, we show theoretically how these effects may be observed in a very different physical system: ultracold atoms in optical potentials. Such observations should also have an important impact on the development of atom optics.

In a periodic potential of the solid, the electronic energy spectrum is known to be in the form of Bloch bands separated by energy gaps [1]. The states in each band are labeled by the so-called crystal wave number k (we consider only one dimensional systems for simplicity). In a weak external electric field, k changes in time and the electrons move periodically within each band (Bloch oscillation). When the field is high enough, tunneling between different bands becomes possible, which is called LZ tunneling [2]. On the other hand, for sufficiently small electric field, LZ tunneling is negligible, and a Bloch band splits into a series of equally spaced energy levels [3], called WS ladders, which are observed in superlattices [4]. The spacing is given by the electric force times the lattice constant, and the corresponding wave functions are localized over a length scale equal to the lattice constant times the band width divided by the ladder spacing.

A periodic potential for the atoms can be generated by two counterpropagating laser beams at the same frequency via the dipole force effect [5]. For a two-level atom of transition frequency ω_0, the optical potential is given by $[\hbar\Omega^2/8(\omega_L - \omega_0)]\cos(2k_Lx)$ where k_L and ω_L are laser wave number and angular frequency, respectively, and Ω is the resonant Rabi frequency proportional to the square root of the laser intensity [6]. We neglect spontaneous scattering, which is valid for sufficiently large detuning from resonance. We consider the case of an accelerating standing wave, which is created when the frequency difference of the two beams varies linearly in time. Then, in terms of scaled dimensionless length $\phi = 2k_Lx$, time $\tau = (4\hbar k_L^2/m)t$, and acceleration $\alpha = m^2a/8\hbar^2k_L^3$, the potential has the form

$$V_0\cos(\phi - \tfrac{1}{2}\alpha\tau^2), \quad \text{with } V_0 = \frac{m}{(2\hbar k_L)^2}\frac{\hbar\Omega^2}{8(\omega_L - \omega_0)}, \tag{1}$$

where the unit of energy is taken as 8 times the photon recoil energy $\hbar^2k_L^2/2m$. In the moving frame of the potential, the atoms feel a constant inertial force, simulating the electric force in the usual electronic experiment.

Bloch bands and rates of LZ tunneling.—The eigenenergy equation for the above potential (1) with $\alpha = 0$ can be solved in terms of the Mathieu functions. We show the energy bands in Fig. 1 for the case of $V_0 = 0.4$. There is one band deep below the potential barriers, which may be regarded as the ground state levels of the potential wells broadened into a band due to tunneling through the barriers. The second band lies about the barrier tops and has a substantial width. The gap above this band is located about the energy $E = 0.5$, corresponding to second order Bragg scattering. The third band has an even wider width, with a tiny gap at $E = 1.125$ corresponding to third order Bragg scattering. We study how the atoms are trapped in the first and second bands and how they are dragged when the potential accelerates.

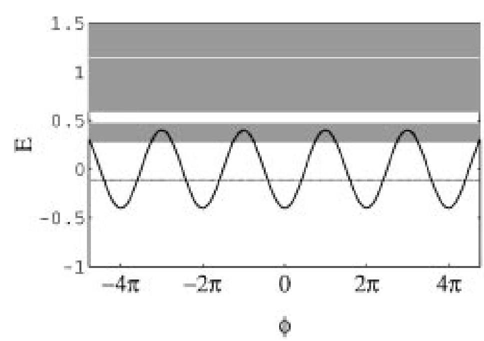

FIG. 1. Potential energy $V_0\cos\phi$ and the Bloch bands for $V_0 = 0.4$.

The LZ tunneling rate across a gap under acceleration α of the potential can be estimated as [2]

$$\gamma = \alpha \exp(-\alpha_c/\alpha), \tag{2}$$

where $\alpha_c = \pi\Delta^2/K$ is the critical acceleration, with Δ the half width of the gap, and $K = n/2$ the wave number of Bragg scattering corresponding to the nth gap. For the gap above the third band, we find $\alpha_c = 1.43 \times 10^{-4}$. Therefore, for the range of accelerations that we consider, $\alpha \geq 0.01$, this gap (and all the higher ones) may be ignored. In other words, the atoms in the third and higher bands move quite freely and are not dragged by the potential. For the gap above the second band we find $\alpha_c = 0.01$, while for the gap below it we find $\alpha_c = 0.23$. Therefore, for accelerations between these critical rates, we expect that only the states in the lowest band are dragged with appreciable probability.

Atomic dragging by an accelerating potential.—We now describe a practical procedure for observing LZ tunneling, which will also serve an important function for the observation of WS ladders to be discussed below. Consider an ensemble of atoms with a Gaussian spread in velocity and uniform distribution in space. We numerically integrate the time-dependent Schrödinger equation to obtain the velocity distribution of the atoms (Fig. 2) at times when the velocity of the potential has been accelerated to $v = 10$ (in units of $2\hbar k_L/m$). The standard deviation of the initial Gaussian distribution was $\sigma = 1.5$, and the potential was suddenly turned on to a strength of $V_0 = 0.4$ at time zero when the acceleration began. We will address the effects of sudden and adiabatic turning on of the potential later.

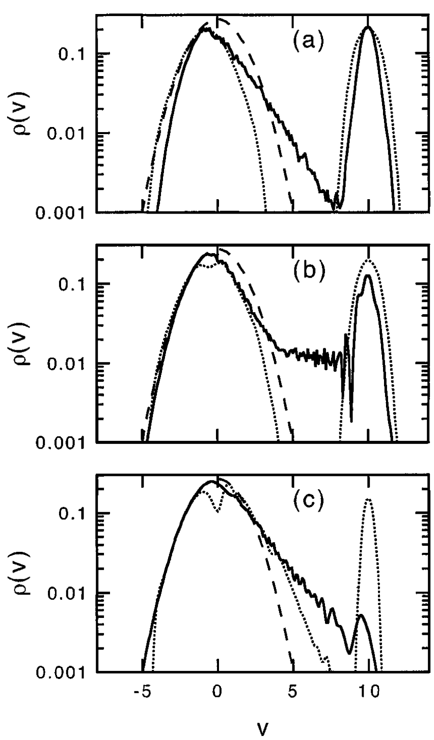

FIG. 2. Velocity distribution of the atoms (solid lines) when the potential has been accelerated to a velocity of $v = 10$ with different accelerations: (a) $\alpha = 0.02$, (b) $\alpha = 0.1$, and (c) $\alpha = 0.3$. The dotted lines are classical results. Dashed lines are for the initial Gaussian distribution.

We identify the states in the dragged peak in Fig. 2(a) (for $\alpha = 0.02$) as being from the first band. We can estimate the velocity distribution in the following approximations: (i) We neglect the tunneling between the wells, so that the velocity distribution of the band is the sum of velocity distributions in the lowest states of the different wells. (ii) The lowest state in a well is taken as the ground state of the harmonic approximation of the well bottom, so that its velocity distribution is given by the Gaussian

$$\exp(-v^2/2\sigma_0^2), \tag{3}$$

where $\sigma_0 = V_0^{1/4}/\sqrt{2} = 0.56$. This is also the distribution of the first band up to a normalization constant, because the velocity distributions for different wells are identical. The peak width estimated this way is very close to the numerical result for the dragged peak shown in Fig. 2(a).

The tail between the original peak and the dragged peak consists of atoms that have tunneled out of the second band and have been left along the way during dragging ("debris"). From a simple model calculation, the probability density of this tail should be given by

$$P_2 \frac{\gamma}{\alpha} \exp\left(-\frac{\gamma v}{\alpha}\right), \tag{4}$$

where P_2 is the initial probability of trapping into the second band, and γ is the rate of LZ tunneling from (2). The log plot of Fig. 2(a) gives a straight line for the tail, confirming the functional form of the above expression. From the slope, the tunneling rate is found to be $\gamma = 0.014$. This gives the critical acceleration via Eq. (4) to be $\alpha_c = 0.0076$, which is somewhat lower than the earlier rough estimate of $\alpha_c = 0.01$.

For higher accelerations, the tail from the second band quickly becomes negligible, but the dragged peak remains the same until α is increased to about 0.1, when it starts to diminish due to appreciable LZ tunneling from the first band. This is seen in Fig. 2(b), where the new tail is now due to atoms lost from the dragged peak along the way. From the slope of this tail, the tunneling rate is found to be about 0.008, and the critical acceleration for tunneling is found to be about 0.25, which is very close to the previous estimate of $\alpha_c = 0.23$. Also, the initial trapping probability in the lowest band P_1 is estimated to be about 0.25 using (4) (with P_2 replaced by P_1). At $\alpha = 0.3$, the dragged peak disappears almost completely [Fig. 2(c)].

We also present the corresponding classical results in Fig. 2 for comparison. It is convenient to consider the problem in the moving frame, where the potential is stationary but is tilted by the inertial force as $V_0 \cos\phi + \alpha\phi$. Dragging is possible for all accelerations below the critical value $\alpha = V_0$ at which the tilted potential ceases to have potential wells. There are two notable differences: (i) There is a gap between the main peak and dragged peak in which the velocity distribution vanishes, because the atoms are either dragged or nondragged because there is no possibility of tunneling in classical mechanics. (ii) For the range of accelerations considered, the line widths of the dragged peaks are wider than those of the quantum results because of the absence of energy quantization.

Atomic trapping in a periodic potential.—For the observation of LZ tunneling and WS ladders (see below) and many other purposes, it is desirable to have as many atoms as possible trapped in the lowest band. We would like to discuss two schemes of trapping and compare their efficiency. Our initial distribution is always a Gaussian in velocity and uniform in position.

The results in Fig. 2 were obtained in the sudden limit where the potential was turned on in a very short time before acceleration. The trapping probability is just the integral of the initial thermal distribution times the velocity distribution in the lowest band. In the Gaussian approximation (3) for the states in the lowest band, the result is

$$P_1 = \frac{1}{\sqrt{2\pi(\sigma^2 + \sigma_0^2)}} \,. \qquad (5)$$

This has the numerical value of 0.25 for the parameters that we used, and is very close to the weight of the dragged peak in Fig. 2.

In the adiabatic limit, all the atoms initially in the first Brillouin zone, $(-\frac{1}{2} < k < \frac{1}{2})$, are trapped in the lowest band [7]. The probability approaches unity when $\sigma \ll 1/2$, and approaches $1/\sqrt{2\pi\sigma^2}$ for σ bigger than half. Therefore, if the initial thermal distribution is wider than the first Brillouin zone, there is not much difference in trapping efficiency between the adiabatic and sudden limits. However, adiabatic turning-on can lead to dramatic improvements when the initial width is narrower. In general, those atoms initially in the nth Brillouin zone, $[(n-1)/2 < |k| < n/2]$, are trapped in the nth band in the adiabatic limit. It is interesting to note that the adiabatic trapping probabilities in the band are independent of the potential strength.

WS ladders.—In Fig. 3 we show the tilted bands at $\alpha = 0.02$ in the moving frame, with the levels of WS ladders (of spacing $2\pi\alpha$) schematically drawn for the lowest two bands. The ladders for higher bands are too short lived to make any physical sense and are therefore ignored completely. The ladder states in the lowest band are essentially localized within individual potential wells. Their broadening due to LZ tunneling at this acceleration is negligible. The ladder states for the second band extend over several periods of the potential, and their broadening (full width at half maximum) due to LZ tunneling can be estimated as $\alpha \exp(-\alpha_c/\alpha)$, where $\alpha_c = 0.01$. The broadening is about 10 times smaller than the spacing between the levels.

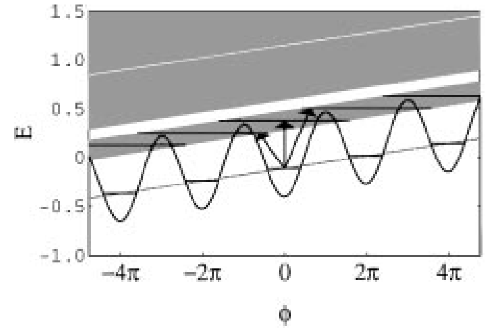

FIG. 3. Tilted bands and WS ladders for $V_0 = 0.4$ and $\alpha = 0.02$. Arrows indicate resonant excitations by an ac modulation of the acceleration.

In order to observe the WS ladders, we add an oscillatory component in the acceleration (simulating an ac electric force), so that the potential has the form $V_0 \cos[\phi - \frac{1}{2}\alpha\tau^2 - \lambda\cos(\omega_m\tau)]$, where ω_m is the modulation frequency and λ the amplitude. This should be able to resonantly excite the atoms in the lowest levels to the ladders in the second band (arrows in Fig. 3). Once the atoms are excited to the higher levels, they are much less likely to be dragged by the wells. The energy structure of the WS ladders is then observed through the number of atoms that are dragged as a function of the excitation frequency. In Fig. 4, we show the calculated probability density at the center of the dragged peak as

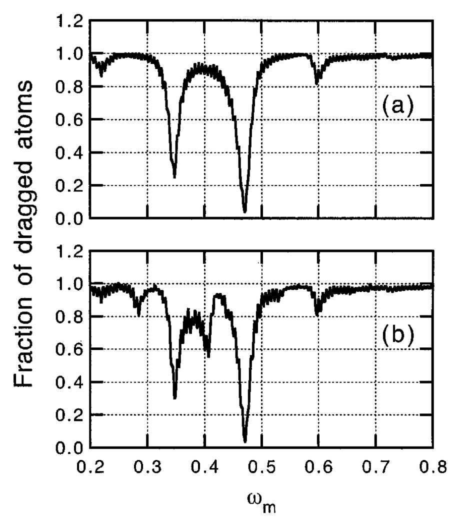

FIG. 4. Fraction of dragged atoms as a function of the excitation energy ω_m with excitation amplitude $\lambda = 0.75\alpha/\omega_m$, with $\alpha = 0.02$. In (a), resonances appear with a spacing $\delta\omega_m = 2\pi\alpha$ of the WS ladders. In (b), the extra resonances are due to an additional (strong) modulation of frequency $\omega_s = 3\pi\alpha$ and amplitude $\lambda_s = 3\alpha/\omega_s$.

a function of the excitation frequency, where the vertical scale is normalized against the number in the absence of excitation. The sequence of equally spaced dips correspond to the resonant excitations to the ladder states, with the spacing equal to that of the WS ladders. Also as expected, the range of frequency showing appreciable resonant excitations is roughly the width of the second band, and this range is centered at a frequency corresponding to the spacing between the centers of the first two bands. The fine scale oscillations of the spectrum are due to finite interaction time ($\tau = 1000$).

An extension of this method can be used to observe *fractional* WS ladders, which were found theoretically [8], but have not yet been observed in experiments. Consider the potential $V_0 \cos[\phi - \frac{1}{2}\alpha\tau^2 - \lambda_s \cos(\omega_s \tau) - \lambda \cos(\omega_m \tau)]$, where λ_s and ω_s are the amplitude and frequency of a strong ac modulation in addition to the original modulation for excitation. If ω_s matches the ordinary ladder spacing by a fractional factor, q/p, the spectrum becomes a *fractional* ladder with a spacing which is p times smaller. Figure 4(b) is for the case with $q = 3$ and $p = 2$. As expected, additional resonances appear at half way between the peaks from the ordinary ladders.

Practical considerations and future directions.—We now discuss the practical feasibility of the proposed experiments for the case of sodium atoms. Our assumed initial velocity spread corresponds to a few photon recoil velocity, which has been routinely achieved using the technique of laser cooling. The range of considered accelerations $\alpha = 0.01-1$ corresponds to $a = 0.74-74$ km/s^2 [9], and the resonance shown in Fig. 4(a) are at 44, 69, 94, and 120 kHz. In order to resolve the ladder spacing, the transition rate of resonant excitation has to be less than $2\pi\alpha$. For $\alpha = 0.02$, this requires the duration of excitation to be longer than 6.4 μs. On the other hand, in order to resolve the natural linewidth (and shape) of the ladders due to LZ tunneling, the required minimum duration of excitation should be about 10 times longer. Since the present experimental capability can allow a duration as long as 200 μs without any interruption (due to laser instability, etc.), it is possible to observe the ladder structure cleanly and in detail. Complicated and interesting behavior of the linewidth as a function of ladder spacing has been predicted [10].

A future interesting direction is to study the effects of noise and dissipation on LZ tunneling [11] and WS ladders. Another direction is to utilize the dragging and excitation technique to prepare atoms in different Bloch bands.

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Note added.—After the submission of this Letter, we learned that Bloch oscillation in a similar atomic system was observed [M. Ben Dahan, E. Peik, J. Reichel, Y. Castin, and C. Salomon, [12]. Also, experimental confirmation of the predicted Wannier-Stark ladders has been carried out [13].

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