Comment on "Nature of Quantum Localization in Atomic Momentum Transfer Experiments"

In a recent Letter, Latka and West [1] claim that quantum suppression of atomic momentum transfer seen in recent experiments [2] is not due to dynamical localization (DL). We contend that their arguments proceed from a misunderstanding of both DL as well as aspects of the experiment. This is compounded by erroneous numerical results which invalidate their conclusions.

DL is a global mechanism for quantum suppression of diffusion characterized by exponentially localized eigenstates or quasienergy states [3]. Even in paradigm systems like the standard map the localization length ξ can fluctuate substantially with the center of mass of the eigenstates, relative to the heuristic estimate. It is only in the limit of an asymptotically large stochasticity parameter that *uniform* DL occurs where the localization length ξ becomes insensitive to the peak position of the eigenstate. Away from this limit, DL is still a valid mechanism as long as the phase space is predominantly chaotic. Apparently, Latka and West misinterpret DL to mean only *uniform* DL.

The experimental initial conditions [2] average the fluctuations in ξ. For certain parameter regimes that are predominantly chaotic, the final momentum distributions are in excellent agreement with the predictions of DL. Even in regimes with small regular regions, the evolution from our initial condition can develop exponential tails which are well described by DL. By contrast, if the overlap of the initial condition with the regular region is substantial, DL is not expected to apply as shown for $\lambda = 3.8$ in the latter paper in Ref. [2].

The role of the momentum boundary in the modulated standing wave is also not understood in Ref. [1]. Resonant kicks occur when the atomic velocity p matches the velocity of the standing wave $\lambda \cos(t)$. Thus, when $p > p_{\max} = \lambda$, the kicks turn off and momentum growth stops with important consequences. At $\lambda = 1.52$, though the classical phase space is chaotic, DL cannot be observed there because ξ exceeds $p_{\max}$, a feature which has to be considered in *any bound system*. In our system, a boxlike momentum distribution results which is shown and discussed in Ref. [2].

We have performed quantum simulations using both space-time integration and Floquet methods [2] for the parameters and initial conditions of Ref. [1]. The Floquet results provide the long-time averaged momentum distributions. The case of $\lambda = 3.0$ shown in Fig. 1 clearly exhibits exponential localization for both the methods we employed. The asymmetry in the line shape is due to the boundary. This is in sharp contrast with the boxlike distribution reported by Latka and West, for the same conditions, which is used to illustrate many of their arguments. The saturated quantum momentum rms width we observe is considerably less than its classical counterpart, which also disagrees with what is shown in Fig. 2 of Ref. [1]. Their numerical results for other λ are also incorrect.

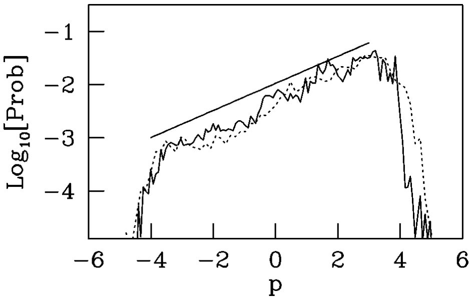

FIG. 1. Final momentum distribution for $k = 0.36$, $\lambda = 3.0$, and $\bar{k} = 0.16$ starting from a minimum uncertainty packet centered at $(q, p) = (2.0, 3.0)$. Both $t = 40T$ (solid line) and the long-time Floquet analysis (dotted line) are shown. The straight line is a fit to the exponential and differs from the naive prediction for the kicked rotor by a factor of 2, which is within expected fluctuations in localization length. Momentum is expressed in the scaled units defined in Ref. [2].

The quantum kicked rotor, the paradigm for DL, has recently been realized experimentally with ultracold atoms [4]. Diffusive growth of momentum is observed until the quantum break time followed by dynamical localization. Though this system is conceptually simpler than the modulated standing wave, it is important to stress that the observed effects are very similar for both systems. Indeed, it is the universal nature of dynamical localization which makes it such an important effect.

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[3] See, for example, articles in *Chaos and Quantum Physics*, edited by M.-J. Giannoni, A. Voros, and J. Zinn-Justin (North-Holland, Amsterdam, 1991).

[4] F. L. Moore, J. C. Robinson, C. F. Bharucha, Bala Sundaram, and M. G. Raizen, Phys. Rev. Lett. **75**, 4598 (1995).